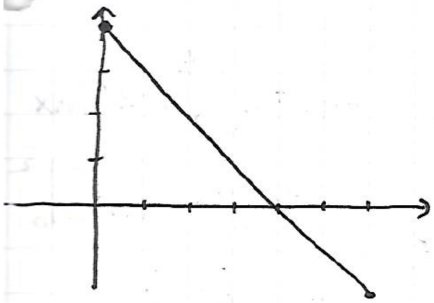
AP Calculus AB

Area Between Two Curves

3) $y = 4 - x$ on $[0, 6]$



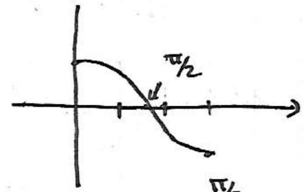
$$\text{Area} = \int_0^4 (4-x)\,dx + \left|\int_4^6 (4-x)\,dx\right|$$

$$= \left[4x - \tfrac{1}{2}x^2\right]\Big|_0^4 + \left|\left[4x - \tfrac{1}{2}x^2\right]\Big|_4^6\right|$$

$$= 8 + |(6-8)|$$

$$= \boxed{10}$$

4) $y = \cos x$ on $[0, \pi]$



$$\text{Area} = 2\int_0^{\pi/2} \cos x\,dx$$

$$= 2\left[\sin x + C\right]\Big|_0^{\pi/2}$$

$$= 2\left[\sin \tfrac{\pi}{2} - \sin 0\right]$$

$$= \boxed{2}$$

5) $$\text{Area} = \int_0^{\pi/4} \left[\sec^2 x - \sin x\right] dx$$

$$= \left[\tan x + \cos x + C\right]\Big|_0^{\pi/4}$$

$$= \left[1 + \tfrac{\sqrt{2}}{2}\right] - \left[1\right]$$

$$= \boxed{\tfrac{\sqrt{2}}{2}}$$

6) $$\text{Area} = 2\int_0^1 \left[x^2 - (-2x^4)\right] dx$$

$$= 2\int_0^1 \left[x^2 + 2x^4\right] dx$$

$$= 2\left[\tfrac{1}{3}x^3 + \tfrac{2}{5}x^5 + C\right]\Big|_0^1$$

$$= 2\left[\tfrac{1}{3} + \tfrac{2}{5}\right]$$

7) TWO Integrals NEEDED BECAUSE "Top" CHANGES

$$\text{Area} = \int_0^1 \left[x - \tfrac{1}{4}x^2\right] dx + \int_1^2 \left[1 - \tfrac{1}{4}x^2\right] dx$$

$$= \left[\tfrac{1}{2}x^2 - \tfrac{1}{12}x^3 + C\right]_0^1 + \left[x - \tfrac{1}{12}x^3 + C\right]\Big|_1^2$$

$$= \left[\tfrac{1}{2} - \tfrac{1}{12}\right] + \left[\left(2 - \tfrac{2}{3}\right) - \left(1 - \tfrac{1}{12}\right)\right]$$

8) TWO INTEGRALS NEEDED SINCE "TOP" & "BOTTOM" SWITCH

$$\text{Area} = \int_{-2}^{0} \left(2x^3 - x^2 - 5x - (-x^2 + 3x)\right) dx$$

$$+ \int_0^2 \left[-x^2 + 3x - (2x^3 - x^2 - 5x)\right] dx$$

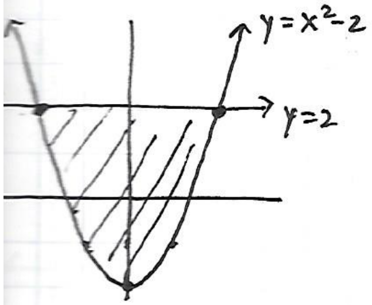
$$= \int_{-2}^{0} \left[2x^3 - 8x\right] dx + \int_0^2 \left[8x - 2x^3\right] dx$$

$$= \left[\tfrac{1}{2}x^4 - 4x^2\right]\Big|_{-2}^{0} + \left[4x^2 - \tfrac{1}{2}x^4\right]\Big|_0^2$$

$$= \left[0 - (8 - 16)\right] + \left[(16 - 8) - 0\right]$$

$$= \boxed{16}$$

9) $y = x^2 - 2$ & $y = 2$



$x^2 - 2 = 2$

$x^2 = 4$

$x = \pm 2$

TOP = 2

BOTTOM = $x^2 - 2$

$$\text{Area} = \int_{-2}^{2} \left[2 - (x^2 - 2)\right] dx$$

$$= \int_{-2}^{2} \left[-x^2 + 4\right] dx$$

$$= \left[-\tfrac{1}{3}x^3 + 4x + C\right]\Big|_{-2}^{2}$$

$$= \left[-\tfrac{8}{3} + 8\right] - \left[\tfrac{8}{3} - 8\right]$$

$$= \frac{32}{3}$$

OR

$$\text{Area} = 2\int_{0}^{2} \left[4 - x^2\right] dx$$

$$= 2\left[4x - \tfrac{1}{3}x^3\right]\Big|_{0}^{2}$$

$$= 2\left[8 - \tfrac{8}{3}\right]$$

$$= \frac{32}{3}$$

10) $y = 7 - 2x^2$; $y = x^2 + 4$

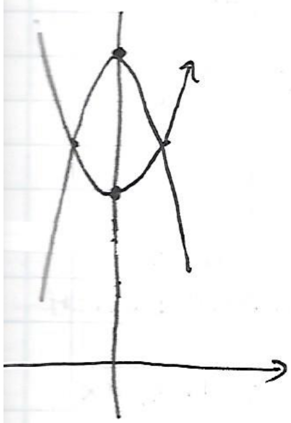
$7 - 2x^2 = x^2 + 4$

$3 = 3x^2$

$x = \pm 1$

TOP = $7 - 2x^2$

BOTTOM = $x^2 + 4$



$$\text{Area} = 2\int_{0}^{1} \left[7 - 2x^2 - (x^2 + 4)\right] dx$$

$$= 2\int_{0}^{1} \left[3 - 3x^2\right] dx$$

$$= 2\left[3x - x^3 + C\right]\Big|_{0}^{1}$$

$$= 2\left[(3 - 1) - 0\right]$$

$$= \boxed{4}$$